

A COMPARATIVE ANALYSIS OF THE EFFECT OF USING DIFFERENT TYPES OF MEMBERSHIP FUNCTIONS (MFs) ON THE PERFORMANCE OF A FUZZY LOGIC CONTROLLER

^aBajeh, O. A., ^bSadiku, J. S. and Emuoyibofarhe O.J

^{A,b} Department of Computer Science, Faculty of Communication & Information Science,
University of Ilorin, Ilorin, Nigeria.

^c Department of Computer Science & Engineering, Ladoké Akintola University,
Ogbomoso, Nigeria.

Email: ^a bajeh_amos@yahoo.com ; bajehamos@unilorin.edu.ng.

^b jssadiku@unilorin.edu.ng. ^c eojustice@justice.com or eojustice@gmail.com

ABSTRACT

Linguistic variables and Linguistic terms/values are fundamental components of a Fuzzy Logic Controller. To a large extent they determine the design of other components (e.g. rule base) and thus the efficiency of the controller as a whole. Linguistic Variables are designed to take Linguistic Terms as values and these Terms are quantified using parameterized Membership Functions (MFs). The MFs yield a certainty value (scaled within the range of $[0, 1.0]$) which indicates the extent to which a value in the Universe of Discourse of a Linguistic variable belongs to a Linguistic Term. Different types of MFs have been designed and used in Fuzzy Logic Controllers and more are still being designed.

*This research work carries out a comparative analysis of the performance of a fuzzy Controller designed with varying MFs while other components are kept constant (i.e. only the MFs are changed). The MFs considered in this work are the **Triangular MF** and the **Gaussian MF**. The choice of these MFs is informed by the fact that Triangular MF has sharp edge, peaking at a point in the Universe of Discourse while the Gaussian MF has a gradual transition edge (no sharp point).*

The performance measures were derived from a Computer simulation of the designed Fuzzy Controller.

Keywords: Fuzzy Logic, Fuzzy Controller, Membership Function (MF), Linguistic Terms, Linguistic Variables, Gaussian MF, Triangular MF.

is gradual and not sharp as in classical set [1]. The smooth transition in fuzzy set is characterized by the use of Membership Functions that gives fuzzy set flexibility in modeling commonly used linguistic expressions or terms such as “the water is hot” or “the temperature is high” [1]. Fuzzy Control, thus, provides a formal methodology for representing, manipulating and implementing a human heuristic knowledge about how to control a system [2]. Some systems complexity gets high so that the control of such system can only be achieved through the use of human heuristic knowledge (fuzzy) rather than mathematical models (conventional control).

The efficient performance of a Fuzzy Controller largely depends on the appropriate choice of Linguistic Variables, Linguistic Terms and **Membership Functions (MFs)** and heuristically generating suitable Linguistic If-Then Rules which will be etched into the Controller's Rule Base for Fuzzy reasoning and control.

In the design of a Fuzzy Logic Controller, having identify the Linguistic Variable(s) and their Linguistic Terms, the Terms are quantified/represented using parameterized MFs. There are different types of parameterized MFs from which a designer of a Fuzzy Logic Controller can choose from. This article involves the study of the possibility of observing an improvement on the performance of a Fuzzy Logic Controller when

designed with a particular MF over another MF.

2. RELATED WORK

Fuzzy Logic has been employed as a methodology for the design of Control Systems. It has served as a good (if not better) alternative methodology for Controller designs. Some of the related works identified in the course of this research work are:

- The Reactor Temperature Controller as reported in [26], where a product of Apronix called FIDE was used to design a fuzzy Controller for the control of the temperature in a Reactor.
- Temperature Control: PID vs Fuzzy Logic as reported in [9] where a Fuzzy Logic version of a PID Home Temperature Control System was reported designed and its advantage over the conventional PID system was highlighted.
- Balancing of an inverted pendulum in a vertical position on a Cart using Fuzzy Control as reported in [2] where a PD Fuzzy Control system was designed. Its performance evaluation was reported done using a simulation of the control system.
- Others as reported in [2] are: Vibration Damping for a Flexible-Link Robot, Rotationally inverted Pendulum, Machine Scheduling and A Fault Detection System for Aircraft. Also reported in [6] is a Fuzzy logic

Speed Control which can also be referred to as Cruise Control.

The Triangular MF was used in the design of the

Controllers cited above. The choice of this MF for the designs was because of the computational simplicity in the use of the MF.

3. DEFINITION OF TERMS

Suppose X is a space of objects and x is a generic element of $X(x \in X)$; a classical conventional set $A(A \subseteq X)$ is defined as a collection of elements or objects $x \in X$ such that each x can either belong or not belong to the set A . This can be mathematically defined by defining a function for each element x in X . This function is referred to as **characteristic function**. The set A can be represented by a set of order pairs: $(x,0)$ or $(x,1)$ which indicates $x \notin A$ or $x \in A$ respectively [1]

Unlike the aforementioned classical set, a fuzzy set expresses the degree to which an element belongs to a set and this degree is represented by a function which is allowed to have values between 0 and 1.0. This function is called The **Membership Function (MF)** and its values denote the degree of membership of the element(s) in a given set.

3.1 Some Terminologies

Definition1: Fuzzy sets and MFs

If X is a collection of objects denoted generically by x , then a fuzzy set A on X is defined as a set of ordered pairs.

$$A = \{(x, \mu_A(x)) / x \in X\}$$

where $\mu_A(x)$ is called the membership function (MF) for the fuzzy set A . The MF maps each element of X to a membership grade/value in a range $[0,1]$ [1]. This MF describes the “certainty” that an element x is the member of the set A . X is referred to as the **Universe of Discourse** or simply **Universe** which is a way to say all the objects in the universe of a particular kind [6]. It may consist of discrete objects (ordered or non-ordered) or continuous space.

Definition 2: Fuzzy Support [1]

The support of a fuzzy set A is the set of the points x in X such that $\mu_A(x) > 0$ i.e.

$$\text{Support}(A) = \{x / \mu_A(x) > 0\}.$$

This is depicted in figure 3.1.

Definition 3: Fuzzy Core [1]

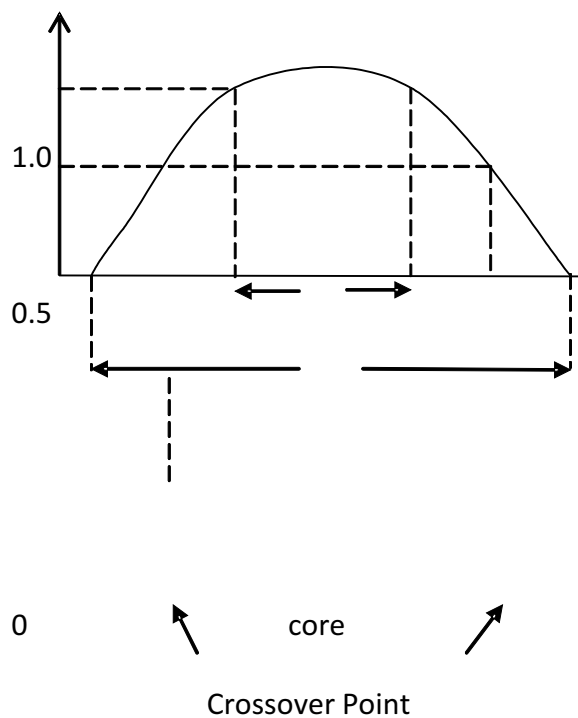
The Core of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$ i.e.

$$\text{Core}(A) = \{x / \mu_A(x) = 1\}.$$
 This is depicted in figure 3.1.

Definition 4: Crossover Points

A Crossover point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$

$$\text{CrossOver}(A) = \{x / \mu_A(x) = 0.5\}.$$
 This is depicted in figure 3.1.



Crossover Point

Support

Figure 3.1

Definition 5: Fuzzy Singleton

Fuzzy set whose support is a single point in X with $\mu_A(x)=1$ is called a fuzzy singleton

3.2 Set Theoretic Operations

Definition 6: Fuzzy set Equality [5]

The fuzzy sets A and B are said to be equal if their MF are equal at every point in the universe of discourse, i.e. $A = B$ if $\mu_A(x) = \mu_B(x) \quad \forall x \in X$.

Definition 7: Fuzzy Complement (Negation) [2]

The complement (“not”) of a fuzzy set A with the MF $\mu_A(x)$ is noted by $\mu_{\bar{A}}(x)$ and has a MF given by $1 - \mu_A(x)$, i.e. $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$.

Definition 8: Fuzzy Set Union (OR)

The union of two fuzzy sets A and B defined over the same universe of discourse X is a new fuzzy set denoted as $A \cup B$ with a MF that represent the maximum degree of relevance between each element and the new fuzzy. [5]. The class of fuzzy union operators use in fuzzy inferences are referred to as T –*Conorm* (or S –*norm*) operators. The following are four T –*Conorm* operators commonly used.

(a) *maximum* : $S(a,b) = \max(a,b) = a \vee b$

(a)

(b) *Algebraic sum* : $S(a,b) = a + b - ab$

(c) *Bounded sum* : $S(a,b) = 1 \wedge (a + b)$

(d) *Drastic sum* : $S(a,b) = \begin{cases} a & \text{if } b = 0 \\ b & \text{if } a = 0 \\ 1 & \text{if } a, b > 0 \end{cases}$

Definition 10: Fuzzy Set Intersection (AND)

The intersection of two fuzzy sets A and B defined over the same Universe of discourse X , is a new fuzzy set denoted as $A \cap B$ with a MF that represent the minimum degree of relevance between each element and the new fuzzy set [5]. The class of fuzzy intersection operators used in fuzzy inferences is referred to as T -norm (Triangular norm). The following are four of the most frequently used T -norm operators

(a) minimum: $T_{\min}(a,b) = \min(a,b) = a \wedge b$

(b) *Algebraic product* : $T_{ap}(a,b) = ab$

(c) *Bounded product* : $T_{bp}(a,b) = 0 \vee (a + b - 1)$

(d) *Drastic product* : $T_{dp}(a,b) = \begin{cases} a & \text{if } b = 1 \\ b & \text{if } a = 1 \\ 0 & \text{if } a, b < 1 \end{cases}$

3.3 One Dimensional MF formulation and Parameterization

Fuzzy sets are completely characterized by their membership functions (MF). In practice the universe of discourse X is a real line R , thus it is impracticable to list all the pairs $(x, \mu_A(x))$ defining the membership function of a fuzzy set. A more concise and convenient way to define a MF is to express it as a mathematical formula [1]. Below are defined some of the most commonly used MFs formulation and on which this work is based:

1. **Triangular MF** [1]: A triangular MF is specified by three parameters $\{a, b, c\}$ where $a < b < c$ which are the x coordinates of the three corners of the underlying triangle as depicted in figure 3.3.1 This function is expressed as follows:

$$\text{triangle } (x : a, b, c) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & c \leq x \end{cases}$$

This can be alternatively stated as:

$$\text{triangl}(x : a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

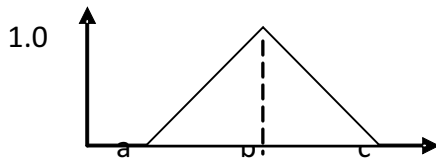


Figure 3.3.1 Triangular MF

2. Gaussian MF [1]:

The Gaussian MF is specified by two parameters $\{c, \sigma\}$ where c represent the MF center and σ determines the MF's width, as depicted in figure 3.3.3. The function is expressed as:

$$\text{gaussian}(x : c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

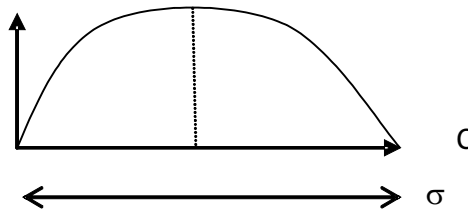


Figure 3.3.3 Trapezoidal MF

3.4. Fuzzy Controller: Generally, a fuzzy system is a static non-linear mapping between its input(s) and output(s). A Fuzzy System has inputs $u_i, i = 1, 2, \dots, n$, outputs $v_j, j = 1, 2, \dots, m$ and other components as shown in figure 3.4 below [2]. The inputs and outputs are 'crisp' (i.e. real number). The **fuzzification** block converts the crisp inputs to fuzzy sets, the **inference mechanism** uses the fuzzy rules in the **rule-base** to produce fuzzy conclusions (implied fuzzy sets) and the **defuzzification** block converts these fuzzy conclusions into the crisp outputs.

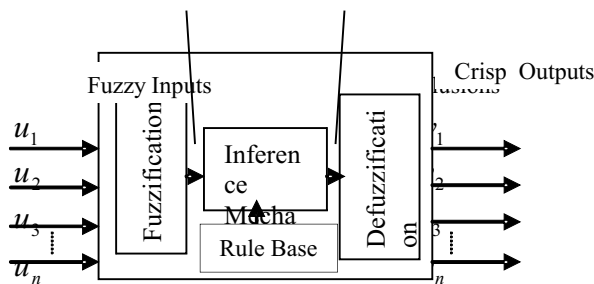


Figure 3.4 Fuzzy Controller

4.1 THE DESIGN OF THE CONTROLLER.

The comparative analysis was conducted on "A Fuzzy Logic Temperature Controller for Preterm Neonate Incubator" [29]. This Controller is in the domain of Process Control Systems. The design of this Controller is as follow:

The Controller is a 2 Inputs, 2 Outputs and 18-MISO rules PD Controller.

a) The Controller's Inputs:

1. **Error in Temperature (e_t):** The difference between the desired value (also known as the reference temperature) and the actual Incubator temperature at a particular time.
2. **Change in error in Temperature ($\dot{e}_t$):** Additional information of whether the temperature is increasing, decreasing or stable will further enhance the ability of the controller to control the temperature. This is change in error and it is derived by taking the derivative of the error in temperature with respect to time

b) The Controller's Outputs:

1. **Voltage ($volt$):** Electrically the voltage across the heating element can determine the quantity of heat generated by the heater. This implies that controlling the voltage across the heater will determine the quantity of heat supply to the Incubator and thereby controlling the temperature
2. **Pressure ($pressure$):** The removal of heat when the temperature goes beyond the reference point is not done by voltage change. High pressure results in high temperature, and low pressure results in low temperature [20]. Taking the advantage of the influence of pressure on temperature, the temperature of the Incubator can be reduced by reducing the pressure in the chamber. This reduces the kinetic energy of the air and allows the flow of fresh and cooler air into the chamber from the surrounding.

Having defined the controller's inputs and outputs the control system was specified as follows:

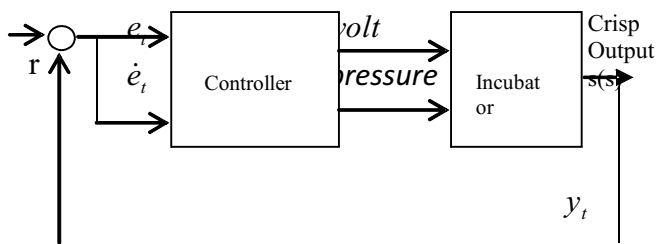
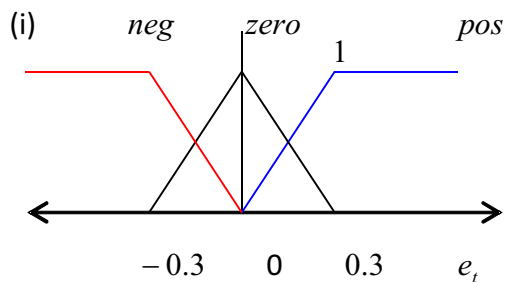


Figure 4.1 fuzzy Temperature Controller

c) Linguistic Variables, Linguistic Value, MFs and Universe Of Discourse.

The controller's inputs and outputs are the systems linguistic variables and they are error in temperature (e_t), change in error ($\dot{e}_t$) change in voltage ($\Delta volt$) and pressure (p_0). Each of these Linguistic Variables is partitioned into three Linguistic Values: *neg*, *zero* and *pos*. Using the Triangular MF there designs are graphically shown in figure 4.2.



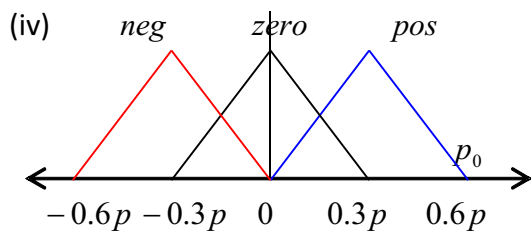
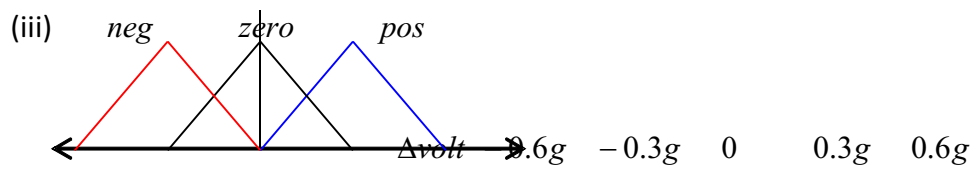
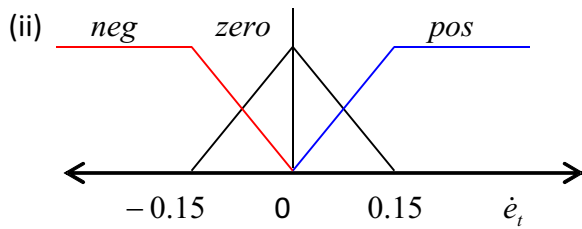
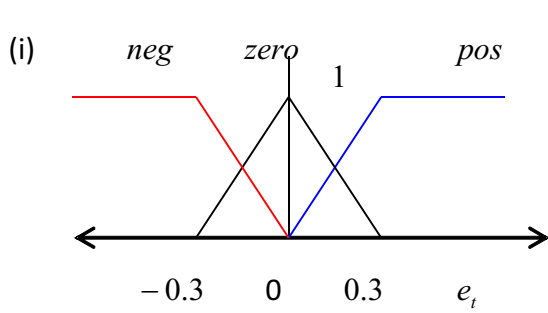


Figure 4.2. Triangular MFs

Using the Gaussian MF the MFs designs are graphically shown in figure 4.3.

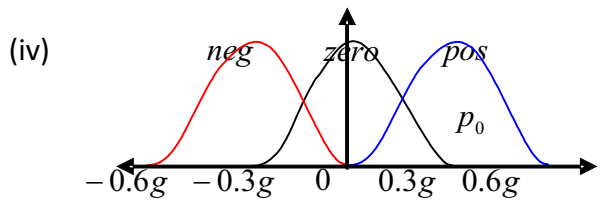
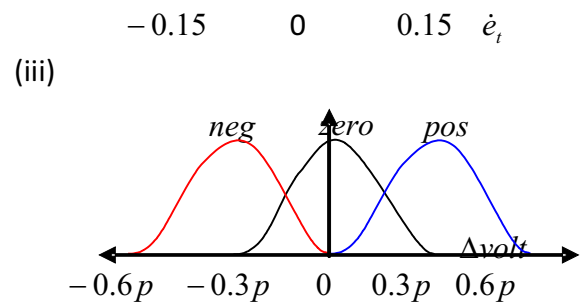
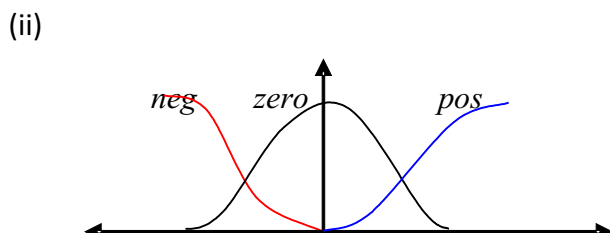
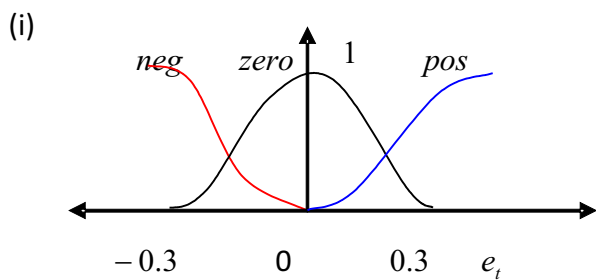


Figure 4.3 MFs defined using Gaussian MFs

(d) The Controller's Fuzzification Interface

The singleton fuzzification is used in the controller. This fuzzification is widely use due to its computational simplicity.

(e) The Controller's Inference Mechanism

The inference mechanism has 2 basic tasks:

Task1: Matching: The fuzzification produces fuzzy sets which the inference mechanism takes as input. The matching takes the following two steps

Step 1: Combine controller input with rule premises: This is the scaling of the inputs fuzzy sets by the singleton fuzzy set input. It simply reduces to computing the membership value of the input fuzzy sets for the given inputs x_1 and x_2 .

Step 2: Determine which rules are on: This step determines the membership values μ_i for the i th rule's premise (i.e. a value for $\mu_{premise}$ for the rule is determined) and it represents the certainty that each rule premise holds for the given input. This is defined as $\mu_{premise} = \mu_i(x_1, x_2) = \mu_{A_{e_1}^i}(x_1) * \mu_{A_{e_2}^i}(x_2)$. Rules with $\mu_i > 0$ are considered to be on or active or fired at the state of the inputs x_1 and x_2 . The operator $*$ is chosen to be the **min** operator.

Task 2: Inference Step: This task computes for the i th rule an implied fuzzy set, B with MF

$\mu_{B_i}(y) = \mu_i * \mu_{B^i}(y)$. The implied fuzzy set B_i is a consequent fuzzy set that specifies the certainty level that the output should be a specific crisp output y on the universe of discourse of the output, when only the i th rule is taken into consideration. In a similar manner the implied fuzzy set B_i for all the rules that are on are computed.

(f) The Controller's Defuzzification Interface.

The commonly used defuzzification method called the Centre of Gravity (COG) is used in this research to compute the crisp value for each of the controller's outputs. The COG defuzzification is determined as:

$$U^{Crisp} = \frac{\sum_{i=1}^R b_i^q \int_{y_q} \mu_{B_q^i}(y_q) dy_q}{\sum_{i=1}^R \int_{y_q} \mu_{B_q^i}(y_q) dy_q} \quad 4.1.7$$

where R in the number of rule, b_i^q is the centre of the area of the membership function B_q^p associated with the implied fuzzy set B_q^i of the i th rule.

4.2 Incubator Temperature Model

The Neonate Incubator is an enclosed chamber made up of a transparent material with apertures for ventilation.

Within the chamber is the heating element which provides heat into the chamber. The voltage output of the temperature controller is set across this heating element to generate heat which is passed into the chamber. In [29] the physics of the temperature of the Incubator was modeled as follows:

$$T_2 = T_1 + \frac{IVt - h_L}{mc} \quad (4.1)$$

$$T_2 = \frac{IVt - mcT_1 + m_i c T_i}{(m_i c - mc)} \quad (4.2)$$

Equation (4.1) models the temperature change when a certain quantity of heat is passed into the chamber by passing voltage across the heating element of the Incubator. Equation (4.2) models the temperature change when the piston is pulled and cooler air flows into the chamber. The two equations (4.1 and 4.2) model the temperature dynamics of the Neonate Incubator in terms of voltage change and pressure change (implemented by using a piston).

5. IMPLEMENTATION

The Fuzzy Controller was programmed in the C++ programming language. This provides some advantage by making it easier to transfer the code directly to an experimental setting for real-time control. MatLab 7.0 was also used to program the controller because it provides a further test for the designed controller. and provides plotting capabilities from which the plotted graph for the rule evaluation and surface graph of the controller as stated in [29] are derived. The pseudocode for the computer program use to develop the controller is :

Fuzzy Controller pseudocode

1. Obtain e and $\dot{e}$ values
(Gets inputs to the fuzzy controller)
2. Compute $mf1[i]$ and $mf2[j]$ for all i, j
(find the values of all MFs given the values of e and $\dot{e}$)
3. Compute $prem[i][j] = \min(mf1[i], mf2[j])$ for all i, j
(Find the value for the premise MF for a given e and $\dot{e}$ using the **min** operation)
4. Compute for all i, j
 $impVolt[i][j] = areaimp(ruleVolt[i][j], prem[i][j])$ and
 $impPressure[j] = areaimp(rulePressure[j], prem[i][j])$ (Find the area under the MFs for possible implied Fuzzy set for the two inputs)
5. Let $num = 0$, $denom = 0$, $num2 = 0$ and $denom2 = 0$
6. For $i = 0$ to 2,
 For $j = 0$ to 2

(Compute change in voltage)

$$\Delta pressure = num2 / denom2$$

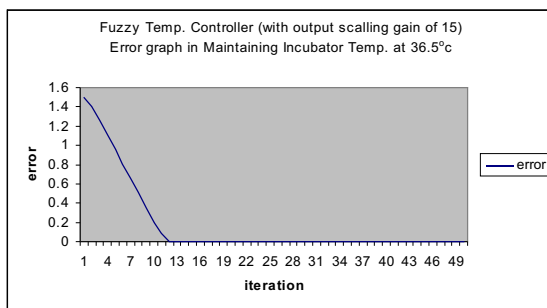
(Compute change in pressure)

8. Output $voltage = voltage + \Delta volt$ and $pressure = pressure + \Delta pressure$
9. Go to step 1.

6. PERFORMANCE EVALUATION

Having developed the computer program to simulate the designed Fuzzy Controller, the model developed for the temperature dynamics of the Incubator was implemented in C++ programming language. These two programs were harmonized to represent a simulation for the entire control system. The data output from the simulation program were used to plot the graphs in figures 6.1 and 6.2. These graphs depict the dynamics of the system which shows how the Incubator temperature is maintained by varying the voltage across the heating element and moving the piston as to stabilize the temperature within the desired point.

(a)



(b)

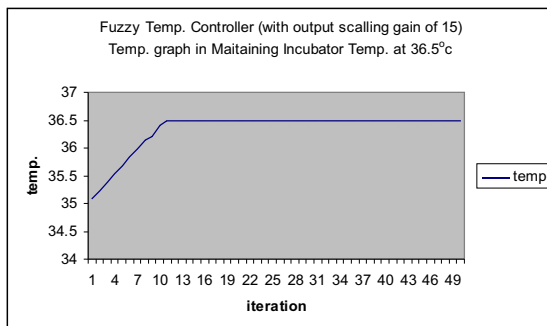
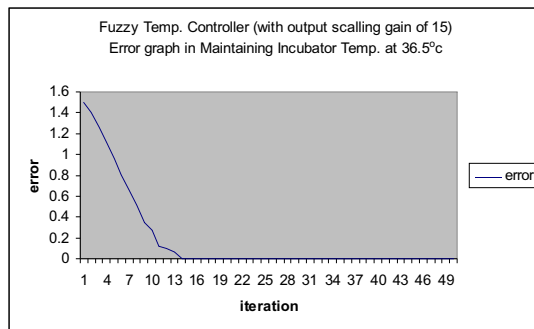


Figure 6.1. The Fuzzy Controller Performance
(controller designed with the TriangularMF)

Figure 6.1 is the graphs showing the performance of the Controller designed with a **Triangular MF**. They show how the Incubator temperature is shot to the desired value of 36.5 and maintained at this value. (a) shows the error decreasing as the

temperature gets stabilized and (b) shows the temperature simultaneously increasing to attain the desired value.

(a)



(b)

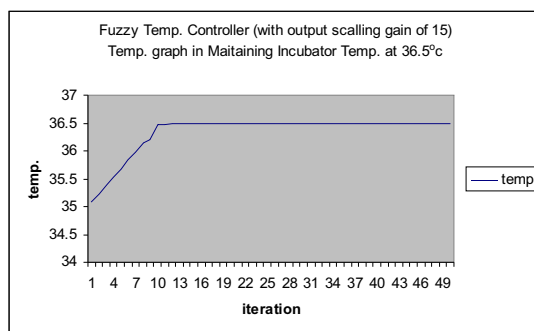


Figure 6.2. The Fuzzy Controller Performance
(controller designed with the Gaussian MF)

Figure 6.2 is the graphs showing the performance of the Controller designed with a **Gaussian MF**. They show how the Incubator temperature is shot to the desired value of 36.5 and maintained at this value. (a) shows the error decreasing as the temperature gets stabilized and (b) shows the temperature simultaneously increasing to attain the desired value.

7 DISCUSSION OF RESULT

The performance evaluation conducted in section 6 from which the graphs in figures 6.1 and 6.2 are derived has shown that the Controller designed meets the closed-loop specification of maintaining the Incubator Temperature at a desired value with allowable rise time, over/under shoot and stability. Figure 6.1 (a) shows how the error in Temperature decreases until it becomes zero, thereby indicating a NORMAL Temperature. Figure 6.1 (b) is the corresponding graph showing how the temperature rises to NORMAL as the error falls to zero. Figure 6.1 shows that the Temperature becomes NORMAL at the 12th iteration of the close-loop system.

Figure 6.2 (a) and (b) shows the same performance as in Figure 6.1 (a) and (b) respectively for the same but designed using a different MF, the Gaussian MF. The graphs in Figure 6.1 look similar to that of Figure 6.2 except that the temperature

becomes NORMAL at the 14th iteration in Figure 6.2, the controller with a **Gaussian MF**.

In conclusion, the performance of the Controller can be said to have little or no significant difference or improvement when the Controller is designed with either the **Triangular MF** or the **Gaussian MF**. The little difference in the number of iterations, 12 against 14 can be attributed to the fact that little extra computation is needed when evaluating the rules using the Gaussian MFs than the Triangular MFs. These serve as contribution to knowledge.

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